Conditional Investment-Cash Flow Sensitivities and Financing Constraints

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Abstract

We study the sensitivity of investment to cash flow conditional on measures of q in an adjustment costs framework with costly external finance. We present a benchmark model in which this conditional investment-cash flow sensitivity increases monotonically with the cost premium for external finance, for firms in a financially constrained regime. Using simulated data, we show that this pattern is found in linear regressions that relate investment rates to measures of both cash flow and average q. We also derive a structural equation for investment from the first order conditions of our model, and show that this can be estimated directly.

1 Introduction

Kaplan and Zingales (1997) emphasize that, in a model with costly external finance, the sensitivity of investment to cash flow need not increase monotonically with the cost premium for external funds. Their result refers to the unconditional correlation between investment and cash flow, and is obtained in a static model with no costs of adjusting the capital stock. In contrast, empirical studies of financing constraints and investment, in the tradition of Fazzari, Hubbard and Petersen (1988), have typically regressed investment rates on measures of both cash flow and q, recognizing the likely presence of adjustment costs. We emphasize the importance of conditioning on measures of q in order to understand the behaviour of the coefficient on cash flow in these specifications.

We study this conditional sensitivity of investment to the availability of internal finance in a benchmark model with quadratic adjustment costs. If external funds and internal funds are perfect substitutes, marginal q provides a sufficient statistic for investment rates in this setting. Following Kaplan and Zingales (1997), we initially consider a model in which new equity is the only source of external finance, and the cost premium increases with the amount of new equity issued. In this case, cash flow may help to explain investment rates, at a given level of marginal q. For firms in the financially constrained regime, where costly new equity is the marginal source of finance, we show that there is a monotonic relationship between the cost premium for external funds and the sensitivity of investment to cash flow, conditional on marginal q.

Provided the net revenue function and the cost premium for external funds satisfy linear homogeneity, we also show that average q equals marginal q in this specification with costly external finance. Hence for constrained firms, we also have a monotonic relationship between the capital market imperfection and investment-

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cash flow sensitivity, conditional on average q. In this model, the relationship between investment rates, average q and cash flow is not linear. Nevertheless we show, using simulated optimal investment data, that estimates of the sensitivity of investment to cash flow conditional on average q obtained in a linear regression framework also vary monotonically with the cost premium for external funds. We find similar results in a model which introduces costly debt as an additional source of external finance, although the equality between marginal q and average q only holds as an approximation in this case.[1]

While this provides a benchmark model of investment with imperfect capital markets in which traditional regressions of investment rates on cash flow and average q could be useful, we also emphasize that the coefficient on cash flow does not have a structural interpretation. The structural first order condition in our model relates investment rates to both average q and an interaction term between average q and a measure of external (new equity) finance. We show that this first order condition can be estimated directly, and the behaviour of the coefficient on cash flow in the traditional regression model can also be understood in relation to the omission of this interaction term from the structural model.

The remainder of the paper is organized as follows. Section 2.1 briefly reviews the sensitivity of investment to cash flow within the framework considered by Kaplan and Zingales (1997). Section 2.2 considers this relationship conditional on marginal q in our basic model with quadratic adjustment costs. Section 2.3 introduces debt finance into our basic specification. Section 3 considers the relationship between marginal q and average q in this setting. Section 4 presents our results using simulated investment data, and derives a structural econometric model that

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[1] Our analysis differs from those of Gomes (2001) and Moyen (2004) principally in that we maintain linear homogeneity. As has been emphasized by Cooper and Ejarque (2003), the relationship between investment, average q and cash flow may be quite different if firms have market power or face decreasing returns to scale, even with no capital market imperfections.

can be estimated directly. Section 5 concludes.

2 Investment-Cash Flow Sensitivities

2.1 Kaplan and Zingales (1997): a static model

In the static setting with no adjustment costs considered by Kaplan and Zingales (1997), the first order condition for the optimal capital stock equates the marginal revenue product of capital to the user cost of capital. If the firm faces a higher cost for using external funds than for using internal funds, the required rate of return on investment financed from external sources will be higher.[2] If this cost premium increases with the level of external funds used, then *ceteris paribus* the optimal capital stock will be lower if the firm is more dependent on external finance. A positive cash flow shock, which increases the availability of low cost internal funds, may then result in higher investment, even if it has no effect on the marginal revenue product of capital. For firms using external funds as their marginal source of finance, this reduces the required rate of return and increases the optimal capital stock. For a given level of capital inherited from the past, investment in the current period is then sensitive to such 'windfall' cash flow shocks.

Kaplan and Zingales (1997) focus on whether investment is *more* sensitive to fluctuations in cash flow for firms that face a *higher* cost premium for external finance. This may not be the case in their model, if the firm's marginal revenue product of capital is sufficiently convex. This possibility is illustrated in Figure 1. There are two firms, identical in every respect except they face different cost premia for external funds, which are reflected in the user cost schedules u_H and u_L. The cost of capital for investment financed internally is u_{INT}, and the marginal

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[2]See Hubbard (1998), for example, for a discussion of why this 'pecking order' assumption may be relevant.

revenue product of capital is denoted MPK.[3] In this case a windfall increase in the availability of internal funds from C to C' increases investment by less for the firm which faces the higher cost of external funds schedule u_H than for the firm with the lower cost premium u_L; that is, $(I'_H - I_H) < (I'_L - I_L)$.

Kaplan and Zingales (1997) thus correctly conclude that there is not necessarily a monotonic relationship between the sensitivity of investment to windfall fluctuations in the availability of internal finance and the slope of the cost of external finance schedule, in a static demand for capital model of this type. However it is not clear what this result tells us about the coefficient on cash flow in an econometric model that also controls for average q. In the rest of this section we consider this conditional investment-cash flow sensitivity in a dynamic investment problem with strictly convex costs of adjustment. This is the basis for the investment-q relation adopted by much of the empirical research in this area, including that presented by Fazzari, Hubbard and Petersen (1988) and by Kaplan and Zingales (1997) themselves.

2.2 A dynamic model with adjustment costs

We study a standard investment problem where the firm chooses investment to maximize the value of its equity V_t given by

$$V_t = E_t \left[\sum_{s=0}^{\infty} \beta^s \left(D_{t+s} - N_{t+s} \right) \right] \tag{1}$$

where D_t denotes dividends paid in period t, N_t denotes the value of new equity issued in period t, $\beta = \frac{1}{1+r} < 1$ is the one-period discount factor, with r the one-period discount rate assumed constant for simplicity, and $E_t[.]$ denotes an expected value given information available at time t.

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[3]The figure is drawn for a given inherited level of the capital stock, so there is a one-to-one association between current investment and the current level of the capital stock.

Dividends and new equity are linked to the firm's net operating revenue Π_t each period by the sources and uses of funds identity

$$D_t - N_t = \Pi_t - \Phi_t \tag{2}$$

where $\Phi_t = \Phi(K_t, N_t)$ represents additional costs imposed by issuing new equity and K_t is the stock of capital in period t. Initially we follow Kaplan and Zingales (1997) in assuming that new equity is the only source of external finance; an extension to a simple specification with debt finance will be considered in the next section. Formally we treat $\Phi(K_t, N_t)$ as a transaction fee that must be paid to third parties when new shares are issued. Less formally we can also think of these costs reflecting differential tax treatments, agency costs, or losses imposed on existing shareholders when the firm issues new shares in markets characterized by asymmetric information.[4] We assume $\Phi(K_t, 0) = 0$, $\Phi_N(K_t, N_t) = \partial\Phi(K_t, N_t)/\partial N_t \geqslant 0$ and $\Phi_K(K_t, N_t) = \partial\Phi(K_t, N_t)/\partial K_t \leqslant 0$.

Following the q literature, we assume $\Pi_t = \Pi(K_t, I_t)$ where $K_{t+1} = (1-\delta)K_t + I_t$, I_t is gross investment in period t (which may be positive or negative), and δ is the rate of depreciation. Net revenue may also depend on stochastic price and productivity terms, which are the source of uncertainty about the future (revenue) productivity of capital. These factors evolve exogenously, and we economize on notation by suppressing their influence on net revenue here. Notice that investment in period t does not contribute to productive capital until period $t + 1$, so that K_t depends only on past investment decisions. With no cost premium for external finance (i.e. $\Phi_t \equiv 0$), this implies that investment in period t is not affected by serially uncorrelated price or productivity shocks, although investment is affected by serially correlated price or productivity shocks that convey information about the (revenue) productivity of capital in period $t + 1$. The dependence

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[4]See, for example, Myers and Majluf (1984).

of net revenue on investment reflects the presence of adjustment costs, which are assumed to be strictly convex in I_t.

The firm maximizes V_t subject to this capital accumulation constraint and to non-negativity constraints on dividends and new equity issues, with shadow values λ_t^D and λ_t^N respectively. The problem can be expressed as

$$V(K_t) = \max_{I_t, N_t} \left\{ \begin{array}{c} \Pi(K_t, I_t) - \Phi(K_t, N_t) \\ +\lambda_t^D \left[\Pi(K_t, I_t) - \Phi(K_t, N_t) + N_t\right] + \lambda_t^N N_t \\ +\beta E_t \left[V\left((1-\delta)K_t + I_t\right)\right] \end{array} \right\} \quad (3)$$

Letting $\mu_t^K = \partial V_t / \partial K_t$ denote the shadow value of inheriting one additional unit of installed capital at time t, the first order condition for optimal investment can be written as

$$-\Pi_I(K_t, I_t) = \frac{\beta E_t\left[\mu_{t+1}^K\right]}{1 + \lambda_t^D} = \frac{\lambda_t^K}{1 + \lambda_t^D} \quad (4)$$

where $-\Pi_I(K_t, I_t) = -\partial \Pi(K_t, I_t)/\partial I_t$ is strictly increasing in the level of investment I_t. If the non-negativity constraint on dividends is not binding ($\lambda_t^D = 0$), this simply equates the marginal cost of investing in an additional unit of capital in period t with the expected shadow value of having an additional unit of capital in period $t+1$, discounted back to its value in period t (see, for example, Abel, 1980). We refer to μ_t^K as the shadow value of capital and to $\lambda_t^K = \beta E_t\left[\mu_{t+1}^K\right]$ as the shadow value of investment at time t; the difference here reflects the timing convention that investment becomes productive with a lag of one period.

Along the optimal path, the evolution of the shadow value of capital is described by the intertemporal condition

$$\mu_t^K = (1 + \lambda_t^D)\Pi_K(K_t, I_t) - (1 + \lambda_t^D)\Phi_K(K_t, N_t) + (1 - \delta)\beta E_t\left[\mu_{t+1}^K\right] \quad (5)$$

where $\Pi_K(K_t, I_t) = \partial \Pi(K_t, I_t)/\partial K_t$.

The first order condition for optimal new share issues implies

$$\lambda_t^D = \frac{\Phi_N(K_t, N_t) - \lambda_t^N}{1 - \Phi_N(K_t, N_t)} \quad (6)$$

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In the case where new shares are issued $(N_t > 0)$ and $\lambda_t^N = 0$, this simplifies to give

$$\frac{1}{1 + \lambda_t^D} = 1 - \Phi_N(K_t, N_t) \tag{7}$$

We now study the relationship between investment and cash flow in this model, conditional on the shadow value of investment (λ_t^K), or marginal q.[5] To be consistent with the specifications adopted in most of the empirical literature, we assume that marginal adjustment costs are linear in the investment rate (I_t/K_t), which restricts adjustment costs to be quadratic. We further assume that $\Phi(K_t, N_t) = \left(\frac{\phi}{2}\right)\left(\frac{N_t}{K_t}\right)^2 K_t$, where ϕ is a parameter that specifies the slope of the cost premium for external finance.[6] In this case $\Phi_N(K_t, N_t) = \phi(N_t/K_t)$, so that the marginal cost premium increases linearly with the amount of new equity issued relative to the size of the firm. In the case where new shares are issued, this gives $\frac{1}{1+\lambda_t^D} = 1 - \phi(N_t/K_t)$.

The first order condition for optimal investment (4) is then depicted in Figure 2, which is adapted from Hayashi (1985), and is drawn for a *given* level of the shadow value of investment (λ_t^K). Again we consider two otherwise identical firms, with the same adjustment cost function, availability of internal funds and shadow value of investment, but subject to different cost schedules for external funds. One firm faces a low cost premium represented by ϕ_L, while the other firm faces a higher cost premium represented by ϕ_H. As before, investment spending exceeding the available level of low cost internal funds has to be financed partly by using more costly external funds. In this financially constrained regime, the firm issues new equity $(\lambda_t^N = 0)$, pays zero dividends $(D_t = 0)$, and λ_t^D is obtained from the

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[5]Marginal q is usually expressed as the ratio of the shadow value of an additional unit of investment (λ_t^K) to the purchase price of a unit of capital. Here we normalize the price of capital goods to unity for simplicity.

[6]In an earlier version of this paper, we also considered a specification with a fixed cost premium per unit of new equity issued (see Bond and Söderbom, 2006).

first order condition for new equity issues (7).[7] For these firms, we consider a 'windfall' increase in the availability of internal funds, which here has no effect on the shadow value (λ_t^K) or marginal q. This would again result in higher investment, by reducing the firm's dependence on external finance and so lowering the shadow value of internal funds (λ_t^D). More importantly, for two otherwise identical firms, a given increase in the availability of internal funds (from C to C') would have a larger effect on investment for the firm which faces a higher cost premium for external finance, conditional on the shadow value of investment or marginal q.

This illustrates the main result of this section. In a model with quadratic adjustment costs and an increasing cost premium for new equity finance, there is a monotonic relationship between this *conditional* sensitivity of investment to fluctuations in cash flow and the slope of the cost schedule for external funds, for otherwise identical firms in the financially constrained regime. Given our timing convention, and the linear homogeneity assumptions that we introduce in section 3 below, the 'windfall' cash flow shock illustrated in Figure 2 corresponds to a serially uncorrelated price or productivity shock; this changes the availability of low cost internal funds, without affecting expected future marginal revenue products of capital, or marginal q. More generally, this is the kind of conditional investment-cash flow sensitivity that is estimated in regression specifications that relate investment rates to measures of cash flow *and* marginal q.

The result is obtained under the assumption that the marginal adjustment cost schedule $-\Pi_I(K_t, I_t)$ is linear in (I_t/K_t), and could clearly be overturned by introducing sufficient curvature into the marginal adjustment cost schedule. However we note that this would be inconsistent with the linear specification of

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[7]Note that the curvature of the $\lambda_t^K (1 - \phi(N_t/K_t))$ schedule in the region where $N_t > 0$ reflects the assumption that, as new shares are issued to finance investment spending above the level that can be funded internally, an increasing proportion of the revenue raised is dissipated by the transaction fee paid to third parties.

the investment-q relationship that has been used to test the null hypothesis of no financing constraints in much of the empirical literature.

2.3 Debt finance

To introduce debt finance into the model, we assume that the firm can issue one-period debt B_t in each period t. This is repaid in the following period, plus an interest charge which we denote by $i_t B_t$. For simplicity, we assume that the interest rate charged may depend on the amount borrowed and on the size of the firm. This gives $i_t = i(K_{t+1}, B_t)$ with $i_B(K_{t+1}, B_t) = \partial i(K_{t+1}, B_t)/\partial B_t \geqslant 0$ and $i_K(K_{t+1}, B_t) = \partial i(K_{t+1}, B_t)/\partial K_{t+1} \leqslant 0$.[8] We further assume that $i(K_{t+1}, 0) = r$ and we restrict borrowing to be non-negative, with a shadow value λ_t^B on this constraint, although these restrictions are not essential.[9]

The firm's optimization problem can now be expressed as

$$
V(K_t, B_{t-1}) = \max_{I_t, N_t, B_t} \left\{ \begin{array}{c} \Pi(K_t, I_t) - \Phi(K_t, N_t) + B_t - (1 + i(K_t, B_{t-1}))B_{t-1} \\ +\lambda_t^D[\Pi(K_t, I_t) - \Phi(K_t, N_t) + B_t - (1 + i(K_t, B_{t-1}))B_{t-1} \\ +N_t] + \lambda_t^N N_t + \lambda_t^B B_t + \beta E_t\left[V\left((1-\delta)K_t + I_t, B_t\right)\right] \end{array} \right\}
$$

The first order conditions for optimal investment (4) and optimal new share issues (6) are unchanged by the introduction of debt in this way. The first order condition for optimal borrowing can be written as

$$
1 + \lambda_t^D + \lambda_t^B = -\beta E_t\left[\mu_{t+1}^B\right] \tag{8}
$$

where $\mu_{t+1}^B = \partial V_{t+1}/\partial B_t$.

If the firm can borrow unlimited amounts at its discount rate r, we have $\mu_{t+1}^B = -(1 + r)$ and (8) implies that $\lambda_t^D = \lambda_t^B = 0$. This gives a special case

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[8] Note that $K_{t+1} = (1-\delta)K_t + I_t$ is known when B_t is chosen in period t. Here we do not explicitly consider the risk of default. For a more rigorous treatment of a dynamic investment problem with risky debt, see, for example, Bond and Meghir (1994).

[9] The model can be extended easily to allow the firm to use some debt in the unconstrained regime, for example if there is a tax advantage to borrowing. This would be reflected in a (tax-adjusted) interest rate below the discount rate r at low levels of debt. The model can also be extended to allow the firm to save as well as borrow, by dropping the non-negativity constraint on B_t, or more generally by introducing a financial asset.

in which borrowing provides a perfect substitute for internal finance, and the firm will never choose to issue costly new shares. Marginal q is a sufficient statistic for investment rates in this case, and there is no 'excess sensitivity' to windfall cash flow shocks that leave marginal q unchanged.

If instead the cost of borrowing is strictly increasing in the level of debt, we again have a financially constrained regime in which additional investment is financed only using costly external sources, and the shadow value of internal funds (λ_t^D) is strictly positive. In this regime, the firm uses an optimal mix of new equity and debt, depending on the cost premium parameters and the risk of being financially constrained in the following period.[10] Given that some new equity is issued, we can again use the first order condition (7) to characterize the behaviour of the shadow value (λ_t^D), as in the analysis of Figure 2 in the previous section. Again we have the result that, conditional on marginal q, the sensitivity of investment to cash flow in the constrained regime will be greater for otherwise identical firms that face a higher cost premium for issuing new equity.

This analysis also suggests that this conditional investment-cash flow sensitivity will vary from zero, in the case where debt provides a perfect substitute for internal funds, to an upper bound given by the case where new equity is the only source of external finance, as we consider increasing the cost premium for borrowing from zero to levels at which debt becomes prohibitively expensive. We investigate whether this conditional cash flow sensitivity varies monotonically with the relevant cost premium parameters for debt using simulated data in section 4 below.

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[10]Note that, since debt has to be repaid in the following period, the firm may be deterred from borrowing this period both by an interest rate above the discount rate, and by the prospect that internal funds may be more scarce next period than they are this period (i.e. by $E_t(\lambda_{t+1}^D) > \lambda_t^D$).

3 Marginal q and average q

The results presented in sections 2.2 and 2.3 condition on the shadow value of investment (λ_t^K) or, equivalently, on marginal q. Fazzari, Hubbard and Petersen (1988), and many subsequent studies, present empirical results for investment models that condition on measures of average q. To relate our results more closely to this empirical literature, we consider the relationship between marginal q and average q in our models with costly external finance.

In a specification with new equity as the only source of external funds, and no cost premium $(\Phi_t \equiv 0)$, restricting the net revenue function $\Pi(K_t, I_t)$ to be homogeneous of degree one in (K_t, I_t) implies equality between marginal q and average q (Hayashi, 1982).[11] With our timing assumption, this gives

$$\lambda_t^K = \beta E_t \left[\mu_{t+1}^K \right] = \frac{\beta E_t[V_{t+1}]}{K_{t+1}} \tag{9}$$

where V_t is the maximized value of the firm. If the firm can borrow as much as it chooses at its discount rate r, the level of debt is indeterminate, and this result generalizes to

$$\lambda_t^K = \beta E_t \left[\mu_{t+1}^K \right] = \frac{\beta E_t[V_{t+1}] + B_t}{K_{t+1}} \tag{10}$$

where V_t is now the maximized value of the firm's equity.

These results imply that, in the absence of financing constraints, the unobserved shadow value of an additional unit of capital can be measured using the observed average value of capital. This allows a measure of marginal q to be constructed using average q, the ratio of the maximized value of the firm to the replacement cost of its inherited capital stock.[12] The numerator of this average

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[11]Sufficient conditions for linear homogeneity of the net revenue function are perfect competition in product and input markets, and constant returns to scale in the production and adjustment cost functions.

[12]We discuss the details for our timing convention in section 4.1 below.

q ratio is often measured using the firm's stock market valuation. Thus, under the null hypothesis of no financing constraints, econometric specifications can in principle condition on marginal q in the benchmark case of a linear homogeneous revenue function and strictly convex adjustment costs.

In the appendix, we show that for the model considered in section 2.3, if we restrict the functions $\Pi(K_t, I_t)$, $\Phi(K_t, N_t)$ and $i(K_t, B_{t-1})$ to be linear homogeneous, this relationship generalizes to

$$\lambda_t^K = \beta E_t \left[\mu_{t+1}^K \right] = \frac{\beta E_t[V_{t+1}] + (1 + \lambda_t^D) B_t}{K_{t+1}} \tag{11}$$

For the special case of the model with new equity as the only source of external finance, this implies that the equality between marginal q and average q expressed in (9) extends to our model with costly external finance, provided the cost premium for new equity also satisfies the linear homogeneity assumption. For the more general model with debt as an additional source of external finance, this suggests that the equality between marginal q and average q expressed in (10) can only be viewed as an approximation that will be accurate for low values of the 'wedge' $\lambda_t^D(B_t/K_{t+1})$. In section 4 we investigate, using simulated data, whether this approximation is sufficiently accurate for our results in section 2 to characterize how the sensitivity of investment to cash flow conditional on average q varies with cost premium parameters for both new equity and debt finance.

4   Results for simulated investment data

4.1   Specification

To generate simulated investment data, we require functional forms for the net revenue function and adjustment costs.

Our net revenue function has the form

$$\Pi(K_t, I_t) = A_t K_t - G(K_t, I_t) - I_t \tag{12}$$

where A_t is a stochastic productivity parameter and $G(K_t, I_t)$ denotes costs of adjustment. The relative price of output and capital goods is assumed to be constant, with both prices implicitly normalized to unity.

We assume a stochastic process for $a_t = \ln A_t$ with two components

$$a_t = a_0 + a_t^P + a_t^T \tag{13}$$

with

$$a_t^P = \rho a_{t-1}^P + u_t. \tag{14}$$

The innovations u_t and a_t^T are drawn independently from homoskedastic Normal distributions, with variances σ_u^2 and σ_T^2 respectively. The log of productivity thus follows a first order Markov process with both persistent and transitory components. The transitory component does not influence the investment decision if the firm faces no cost premium for external finance, but does affect the availability of internal funds to finance investment spending. We choose parameters $a_0 = -1.7107$, $\rho = 0.8$, $\sigma_u^2 = 0.0225$, and $\sigma_T^2 = 0.0375$, giving serial correlation in a_t equal to 0.5.

We assume a standard functional form for adjustment costs

$$G(K_t, I_t) = \frac{b}{2} \left(\frac{I_t}{K_t} - \delta - e_t \right)^2 K_t \tag{15}$$

which is strictly convex in I_t and homogeneous of degree one in (K_t, I_t). The rate of depreciation is set to $\delta = 0.15$ and e_t is a mean zero adjustment cost shock, distributed as $e_t \sim iid \ N(0, \sigma_e^2)$ with $\sigma_e^2 = 0.0016$.[13] Adjustment costs are

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[13] A similar specification for adjustment costs was suggested by Summers (1981). The presence of adjustment cost shocks is one way of ensuring that the linear relationship between investment rates and average q expressed in equation (17) below does not hold exactly.

minimized by setting net investment to zero on average. Since there is also no trend in the productivity process, this generates optimal choices for investment, capital and output with no systematic trends.

4.1.1 Perfect capital markets

With no cost premium for external funds, this gives a convenient linear functional form for the first order condition for investment (4)

$$\frac{I_t}{K_t} = \left(\delta - \frac{1}{b} \right) + \frac{1}{b} \left(\beta E_t[\mu_{t+1}^K] \right) + e_t \tag{16}$$

where, as noted earlier, $\beta E_t[\mu_{t+1}^K]$ is marginal q given our timing assumption that current investment becomes productive in period $t+1$. Using (10), this can be written as

$$\frac{I_t}{K_t} = \left(\delta - \frac{1}{b} \right) + \frac{1}{b} Q_t + e_t \tag{17}$$

with average q measured as

$$Q_t = \frac{\beta E_t[V_{t+1}] + B_t}{K_{t+1}} = \frac{V_t - X_t + B_t}{K_{t+1}}, \tag{18}$$

where $X_t = \Pi_t - \Phi_t - (1 + i_t)B_{t-1} + B_t$. The adjustment cost parameter b is set to 5, giving a coefficient on average q of 0.2 under the null of no financing constraints. The discount rate r used to generate the simulated investment data is set to 0.04, giving a discount factor β of 0.9615.

Given that the net revenue function (12) is homogeneous of degree one in (K_t, I_t), the firm's value maximization problem would have no unique solution in the absence of strictly convex adjustment costs. We choose parameters for the productivity process such that, on average, the firm would not want to expand or to contract in the absence of adjustment costs. The numerical optimization procedure we use to generate the simulated investment data is described in an appendix.[14]

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[14]This is available at www.nuffield.ox.ac.uk/General/Members/Bond.aspx

4.1.2 Costly external finance

To extend this analysis to include a cost premium for new equity, we use the increasing cost schedule

$$\Phi(K_t, N_t) = \left(\frac{\phi}{2}\right)\left(\frac{N_t}{K_t}\right)^2 K_t \tag{19}$$

that was considered in section 2.2. Setting $\phi = 0$ gives the baseline case in which new equity is a perfect substitute for internal funds, and the investment equation (17) is correctly specified. Setting $\phi > 0$ gives cases in which issuing new equity is more costly than using internal finance, and the investment spending of firms that are issuing new equity is financially constrained in the sense described in section 2.2. We choose values of ϕ and parameters of the productivity process to ensure that a non-negligible proportion of the observations in our simulated datasets are in the constrained regime with $N_t > 0$.

To extend this analysis to include a cost premium for debt, we use the increasing interest rate schedule

$$i(K_{t+1}, B_t) = i + \eta\left(\frac{B_t}{K_{t+1}}\right) \tag{20}$$

where i is the interest rate at zero borrowing, and $\eta \geqslant 0$ is a parameter which allows the interest rate to increase with the debt-assets ratio. We also restrict borrowing to be non-negative. The baseline case sets $i = r$ and $\eta = 0$, in which case there is no cost premium for debt, and the firm is not financially constrained (regardless of the value of ϕ). Provided we have $\phi > 0$ and $i = r$, $\eta > 0$ then implies that external finance is more costly than internal finance, and the investment spending of firms that are using external finance is financially constrained.[15]

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[15] If we have $i < r$ and $\eta > 0$, firms in the unconstrained regime will also choose to borrow, but only firms in the constrained regime will issue new shares.

We consider the behaviour of the estimated coefficients on both average q and the cash flow variable in a standard 'excess sensitivity' test specification[16]

$$\frac{I_t}{K_t} = \left(\delta - \frac{1}{b}\right) + \frac{1}{b}Q_t + \gamma\left(\frac{C_t}{K_t}\right) + e_t \qquad (21)$$

which introduces the ratio of cash flow to capital as an additional explanatory variable in the model (17) derived under the null of no financing constraints. In our most general specification, which includes debt, cash flow (C_t) is measured as $A_t K_t - G(K_t, I_t) - \Phi_t - i_t B_{t-1}$, while output (Y_t) is measured as $A_t K_t$. The null hypothesis $\gamma = 0$ corresponds to the case with no (relevant) cost premium for external funds. More generally, the coefficient γ estimates the sensitivity of investment to cash flow conditional on average q.

This simple linear specification imposes the restriction that this conditional investment-cash flow sensitivity is common to all the observations in the sample. When firms face a cost premium for external finance, this linear model is certainly mis-specified; we know that the conditional sensitivity of investment to cash flow should be different for observations in the constrained and unconstrained regimes. Our analysis using these simulated datasets will thus indicate whether our theoretical results on the monotonic relationship between conditional investment-cash flow sensitivity and the cost premium for external funds *for observations in the constrained regime* is useful for understanding the behaviour of the estimated conditional investment-cash flow sensitivity obtained from these simple investment regressions. In specifications where firms choose to use costly debt, this will also indicate whether average q approximates marginal q sufficiently well for our thoretical result, conditioning on marginal q, to be useful.

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[16] See, for example. Fazzari, Hubbard and Petersen (1988).

4.2 Results

We generate simulated panel datasets for samples with 2000 firms observed for 14 periods, and consider results for the augmented investment-q specification given in (21).[17] The mean of the simulated average q variable is close to one, the mean of the investment rates is close to 0.15, the rate of depreciation, and there are no systematic trends in the capital stocks or other measures of firm size.

Table 1 presents results for models in which new equity is the only source of external finance. Here the equality between marginal q and average q expressed in (9) holds exactly. Column (i) uses simulated data where there is no cost premium for external finance. Columns (ii) - (iv) consider increasing the cost premium parameter (ϕ) for new equity finance.

We report OLS estimates of the inear regression specification (21). For the data generated under the null hypothesis of no financing constraints (column (i)), we estimate the coefficient on the cash flow variable to be insignificantly different from zero, and the coefficient on average q to be insignificantly different from the reciprocal of the adjustment cost parameter (i.e. $1/b = 0.2$; see (17)). For the data generated with a cost premium for new equity finance (columns (ii) - (iv)), we estimate the coefficient on cash flow to be positive and significantly different from zero. Moreover we find that this estimated coefficient increases monotonically as issuing new equity becomes more costly. These simple estimates of the sensitivity of investment to cash flow conditional on average q thus behave in line with our result for the behaviour of this conditional investment-cash flow sensitivity in the constrained financial regime.[18]

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[17]The generated data has the expected time series properties for a model with a linear homogeneous net revenue function, so that in the absence of adjustment costs firms would have no optimal size (see, for example, Lucas, 1967). The logs of the firm value and capital stock series are integrated of order one, while the investment rates and average q series are integrated of order zero, indicating that firm value and capital stocks are cointegrated in this framework.

[18]We can also notice that the estimated coefficient on average q falls monotonically as the cost

Table 2 presents results for models in which the firm can both issue new equity and borrow. We fix the cost premium parameter for new equity at the same value used in column (iv) of Table 1, and we fix the intercept parameter in the interest rate schedule (20) to equal the discount rate (i.e. we have $\phi = 4$ and $i = r = 0.04$). In column (i), the firm can borrow as much as it chooses at this fixed interest rate, giving another specification in which there are no binding financing constraints. In columns (ii) - (iv), the interest rate increases with the amount borrowed, and we explore the effects of varying the slope (η) of this interest rate schedule.

In column (i), where model (17) is correctly specified, we again find that the estimated coefficient on the cash flow variable is insignificantly different from zero, and the estimated coefficient on average q is insignificantly different from $1/b$. In column (iv), borrowing is sufficiently expensive that most firms have very little debt, in which case average q approximates marginal q very well (see (11)), and the estimated coefficients are very similar to those in column (iv) of Table 1. Interestingly, we again find that the estimated coefficient on the cash flow variable increases monotonically, from (essentially) zero to this (approximate) upper bound (given $\phi = 4$), as the cost premium parameter (η) describing the slope of the interest rate schedule increases.

Table 3 shows that a similar pattern is found when we consider increasing the cost premium parameters for both sources of external finance. If we think that 'more severe' capital market imperfections are likely to be reflected in higher cost premia for both new equity and debt, this variation is perhaps the most useful case to consider.

These results suggest that estimates of the sensitivity of investment to cash flow conditional on average q obtained from these simple regression models reflect

premium for external finance increases, both here and in Tables 2 and 3.

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the behaviour of the sensitivity of investment to cash flow conditional on marginal q for observations in the constrained financial regime. This is found despite the mis-specification of these linear models when some firms are financially constrained and others are not, and despite the wedge between average q and marginal q which is introduced by costly debt finance of the type we have considered here. At least in our benchmark specifications, with linear homogeneity, quadratic adjustment costs, and increasing marginal cost premia for both new equity and debt, we find that this estimated conditional investment-cash flow sensitivity increases monotonically with the cost premia for external funds.

4.3   A structural investment model with costly external finance

As we have emphasized, these linear regressions of investment rates on average q and cash flow are not correctly specified models when some firms are financially constrained, and the coefficient on the cash flow term does not have a structural interpretation. As a result, this coefficient varies with other model parameters, such as the persistence in the productivity process, and not only with the cost premia for external finance. In this section we derive an investment equation from the first order condition for optimal investment (4) in our model with costly external finance, and show that this structural model can be estimated directly. This also provides further insight into the behaviour of the coefficient on cash flow in the traditional 'excess sensitivity' regressions.

Combining the first order condition for investment (4) with the first order condition for new shares (6), and using the form of the cost premium for new equity (19) as in (7), gives the condition

$$-\Pi_I\left(K_t, I_t\right) = \beta E_t[\mu_{t+1}^K]\left(1 - \phi\left(\frac{N_t}{K_t}\right)\right) \tag{22}$$

Using the forms of the net revenue function (12) and the adjustment cost function (15) then gives

$$\frac{I_t}{K_t} = \left(\delta - \frac{1}{b}\right) + \frac{1}{b}\left(\beta E_t[\mu_{t+1}^K]\right) - \frac{\phi}{b}\left(\beta E_t[\mu_{t+1}^K]\left(\frac{N_t}{K_t}\right)\right) + e_t \qquad (23)$$

which reduces to (16) when the cost premium parameter for new equity $\phi = 0$, or when no new shares are ever issued. As expected marginal q, as conventionally defined for the case of perfect capital markets (i.e. $\beta E_t[\mu_{t+1}^K]$ given our timing assumptions), is not a sufficient statistic for investment rates in the model with an increasing cost premium for external funds. The additional term is an interaction between marginal q and new equity.[19] This interaction term has a negative coefficient, consistent with the result illustrated in Figure 2 that, at a given level of marginal q, firms using high cost external finance will choose lower investment rates than firms with sufficient low cost internal funds to finance all their investment spending.

We can also use (7) to eliminate the $(1 + \lambda_t^D)$ term from the right hand side of (11), giving

$$\lambda_t^K = \beta E_t\left[\mu_{t+1}^K\right] = \frac{\beta E_t[V_{t+1}]}{K_{t+1}} + \frac{(B_t/K_{t+1})}{1 - \phi(N_t/K_t)} \qquad (24)$$

Given linear homogeneity, we can now use (24) to substitute out the unobserved marginal q terms in (23), giving

$$\frac{I_t}{K_t} = \left(\delta - \frac{1}{b}\right) + \frac{1}{b}Q_t - \frac{\phi}{b}\left[\left(Q_t - \frac{B_t}{K_{t+1}}\right)\left(\frac{N_t}{K_t}\right)\right] + e_t \qquad (25)$$

where average q is again given by (18). This model can be estimated directly, given data on investment rates, average q, debt, and the value of new shares issued. The coefficients estimated are all structural parameters of the adjustment cost function or the cost premium function for new equity, and the error term is

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[19]Note that, in our model with costly external finance, firms in the unconstrained regime do not issue new shares, so that this interaction term is automatically zero for observations in the unconstrained regime.

again the stochastic shock to the rate of investment at which adjustment costs are minimized. Notice that the debt cost premium parameters are not identified from this specification.

Table 4 presents 2SLS estimates of model (25) using lagged average q, lagged debt and new equity terms, and current output as instrumental variables.[20] The four columns use the same simulated datasets that were used in Table 3, with values of the cost premium parameter ϕ increasing from zero to 4. The true values of the coefficient $-\phi/b$ on the interaction term are thus zero in column (i), -0.2 in column (ii), -0.4 in column (iii) and -0.8 in column (iv). The estimated coefficients on the linear average q terms are close to their true value of 0.2 in all four cases. In column (i), the estimated coefficient on the interaction term is not significantly different from zero, correctly indicating that the firms in this sample do not face a cost premium for new equity. In each of columns (ii) - (iv), the estimated coefficient on the interaction term is significantly different from zero, and close to its true value.

This structural model helps to explain the behaviour of the coefficient on the cash flow variable in the OLS estimates of the linear regression specification (21). Compared to the correctly specified structural model, the linear model omits a relevant explanatory variable, the interaction term, and includes an additional explanatory variable, cash flow, which is correlated with the omitted variable. Conditional on average q, firms with higher cash flow are less likely to issue new shares. The partial correlation between the cash flow variable and the omitted interaction term, and the coefficient on this omitted variable, are both negative, consistent with the positive coefficient on the cash flow variable when we estimate

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[20]Both the explanatory variables in (25) are correlated with the error term (e_t), as the use of external finance depends in part on the realization of the adjustment cost shock. Given our timing assumptions, lagged average q, lagged debt and new equity terms, and current output are orthogonal to serially uncorrelated adjustment cost shocks.

the mis-specified linear model. The coefficient on the omitted interaction term increases in absolute value with the cost premium parameter (ϕ) for new equity, consistent with the monotonic increase in the coefficient on cash flow as issuing new equity becomes more expensive (Table 1 and Table 3). Inspection of our simulated data suggests that as we increase the cost premium parameter (η) for debt for a fixed value of ϕ, the negative partial correlation between cash flow and the omitted variable becomes stronger, consistent with the monotonic increase in the coefficient on cash flow as borrowing becomes more expensive (Table 2).

Our results using simulated data also suggest that estimation of this kind of structural model may be a promising direction for empirical research on corporate investment and financing constraints, although the assumption of linear homogeneity, and the conditions needed to measure average q using stock market valuations, may still be unduly restrictive.[21] Hennessy, Levy and Whited (2007) present empirical estimates of a similar model using data for US firms.

5   Conclusions

Following Fazzari, Hubbard and Petersen (1988), a large empirical literature sought to investigate the impact of financing constraints on company investment by regressing investment on q and cash flow. In contrast to Kaplan and Zingales (1997), and much of the subsequent debate,[22] we emphasize the importance of conditioning on measures of q in order to understand the behaviour of the coefficient on cash flow in these specifications.

We present a benchmark specification, for the case of quadratic adjustment costs and an increasing cost premium for new equity finance, in which the sensitiv-

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[21]See, for example, Cooper and Ejarque (2003) and Bond and Cummins (2001).

[22]See, for example, Cleary (1999), Fazzari, Hubbard and Petersen (2000) and Kaplan and Zingales (2000).

ity of investment to cash flow, conditional on marginal q, increases monotonically for firms in the financially constrained regime, as the cost premium parameter for issuing new equity increases. For linear homogeneous functional forms, the same result is shown to hold conditional on average q. Introducing debt as a second source of external finance into the model, with an increasing cost of borrowing, does not change our result conditional on marginal q, but introduces a wedge between marginal q and average q.

Estimating linear regressions that relate investment rates to measures of both cash flow and average q, using simulated data, we nevertheless find a monotonic relationship between these estimates of conditional investment-cash flow sensitivity and the cost premium parameters for both sources of external finance. Although the relationship between cash flow and investment, conditional on average q, is certaintly different for observations in the constrained and unconstrained regimes, these simple estimates of the conditional investment-cash flow sensitivity are consistent with our theoretical results.

We also derive a structural investment equation from the first order conditions of our model, and show that this can be estimated directly. The structural model relates investment rates to average q and an interaction term between average q and new share issues. The behaviour of the estimated coefficient on the cash flow term in the linear regression models can be interpreted in relation to the omission of this interaction term from the correctly specified structural model.

There are several good reasons why regressions of investment rates on average q and cash flow may not provide reliable evidence about the role of capital market imperfections. Even in the absence of financing constraints, marginal q may not be a sufficient statistic for investment if fixed adjustment costs are im-

portant,[23] average q may be a poor proxy for marginal q if firms have market power,[24] or may just be very poorly measured, for example as a result of share price bubbles.[25] We simply note that the non-monotonic relationship between unconditional investment-cash flow sensitivity and the cost premium for external finance, highlighted by Kaplan and Zingales (1997), has little relevance for evaluating this line of research.

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[23]Caballero and Leahy (1996).

[24]Hayashi (1982) and Cooper and Ejarque (2003).

[25]Erickson and Whited (2000) and Bond and Cummins (2001).

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Appendix: Marginal q and average q with costly external finance

We consider the most general model set out in section 2.3, and assume that the functions $\Pi(K_t, I_t)$, $\Phi(K_t, N_t)$ and $i(K_t, B_{t-1})$ are each homogeneous of degree one in their arguments. This implies that the value function $V_t(K_t, B_{t-1})$ is also homogeneous of degree one, and can be written as

$$V_t(K_t, B_{t-1}) = \nu_t(B_{t-1}/K_t)K_t$$

This allows us to write

$$\mu_t^K = \frac{\partial V_t}{\partial K_t} = -\left(\frac{\partial \nu_t(B_{t-1}/K_t)}{\partial(B_{t-1}/K_t)}\right)\left(\frac{B_{t-1}}{K_t}\right) + \nu_t\left(\frac{B_{t-1}}{K_t}\right)$$

and

$$\mu_t^B = \frac{\partial V_t}{\partial B_{t-1}} = \left(\frac{\partial \nu_t(B_{t-1}/K_t)}{\partial(B_{t-1}/K_t)}\right)$$

so that

$$\mu_t^K = -\mu_t^B\left(\frac{B_{t-1}}{K_t}\right) + \left(\frac{V_t}{K_t}\right)$$

and hence

$$\beta E_t\left[\mu_{t+1}^K\right] = -\beta E_t\left[\mu_{t+1}^B\right]\left(\frac{B_t}{K_{t+1}}\right) + \frac{\beta E_t\left[V_{t+1}\right]}{K_{t+1}}$$

noting that $K_{t+1} = (1-\delta)K_t + I_t$ is known at time t.

Using the first order condition (8) to substitute for $-\beta E_t\left[\mu_{t+1}^B\right]$ then gives

$$\beta E_t\left[\mu_{t+1}^K\right] = (1+\lambda_t^D)\left(\frac{B_t}{K_{t+1}}\right) + \lambda_t^B\left(\frac{B_t}{K_{t+1}}\right) + \frac{\beta E_t\left[V_{t+1}\right]}{K_{t+1}}$$

Noting that $\lambda_t^B B_t = 0$ gives

$$\beta E_t\left[\mu_{t+1}^K\right] = \frac{\beta E_t[V_{t+1}] + (1+\lambda_t^D)B_t}{K_{t+1}}$$

as stated in section 3, equation (11).

Table 1. Excess Sensitivity Tests: Model with Costly New Equity, No Debt

	(i)	(ii)	(iii)	(iv)
	$\phi = 0$	$\phi = 1$	$\phi = 2$	$\phi = 4$
Q_t	0.1978	0.1927	0.1830	0.1724
	(.0024)	(.0023)	(.0022)	(.0021)
$\frac{C_t}{K_t}$	0.0071	0.0374	0.0666	0.1053
	(.0064)	(.0059)	(.0057)	(.0054)
R^2	0.34	0.37	0.39	0.43

Sample size: $N = 2000$ $T = 14$ Observations $= 28,000$. OLS estimates. A constant is included in all specifications. Standard errors in parentheses.

Table 2. Excess Sensitivity Tests: Costly New Equity & Costly Debt

	(i)	(ii)	(iii)	(iv)
	$\phi = 4$	$\phi = 4$	$\phi = 4$	$\phi = 4$
	$\eta = 0$	$\eta = 0.25$	$\eta = 1.0$	$\eta = 20$
Q_t	0.2000	0.1888	0.1788	0.1724
	(.0024)	(.0022)	(.0022)	(.0021)
$\frac{C_t}{K_t}$	0.0045	0.0507	0.0734	0.1077
	(.0062)	(.0058)	(.0057)	(.0054)
R^2	0.35	0.39	0.38	0.42

See Table 1 for notes.

Table 3. Excess Sensitivity Tests: Costly New Equity & Costly Debt

	(i)	(ii)	(iii)	(iv)
	$\phi = 0$ $\eta = 0$	$\phi = 1$ $\eta = 0.25$	$\phi = 2$ $\eta = 1.0$	$\phi = 4$ $\eta = 20$
Q_t	0.2013 (.0023)	0.1905 (.0023)	0.1868 (.0023)	0.1743 (.0021)
$\frac{C_t}{K_t}$	0.0022 (.0061)	0.0384 (.0061)	0.0527 (.0058)	0.1084 (.0054)
R^2	0.35	0.36	0.38	0.43

See Table 1 for notes.

Table 4. Structural Model Estimates

	(i)	(ii)	(iii)	(iv)
	$\phi = 0$ $\eta = 0$	$\phi = 1$ $\eta = 0.25$	$\phi = 2$ $\eta = 1.0$	$\phi = 4$ $\eta = 20$
Q_t	0.2075 (.0040)	0.2016 (.0021)	0.1994 (.0022)	0.2034 (.0023)
$\left(Q_t - \frac{B_t}{K_{t+1}}\right) \times \frac{N_t}{K_t}$	-0.0220 (.0198)	-0.2355 (.0463)	-0.4057 (.0511)	-0.8049 (.0564)
p	0.94	0.95	0.33	0.52

Sample size: $N = 2000$ $T = 14$ Observations $= 28,000$

Two stage least squares estimates. A constant is included in all specifications. Standard errors in parentheses.

Instrumental variables: Q_{t-1}, $\left(\frac{B_{t-1}}{K_t}\right)$, $\left(\frac{N_{t-1}}{K_{t-1}}\right)$, $\left(\frac{Y_t}{K_t}\right)$

p is the p-value of the Sargan test of over-identifying restrictions

Figure 1

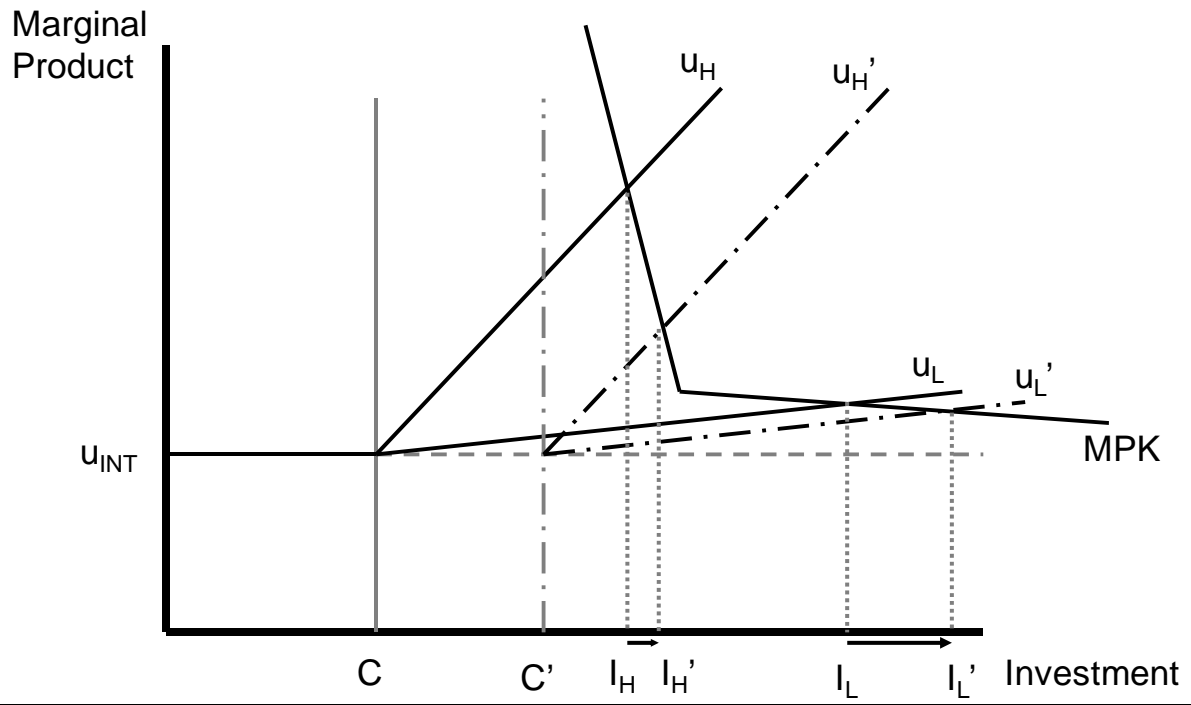**Unconditional Investment-Cash Flow Sensitivity in a Static Model with Convex MPK**

Figure 2

Conditional Investment-Cash Flow Sensitivity in a Dynamic Model Based on Adjustment Costs